Comparison of magnetic field models to magnetospheric cusp positions observed by the Polar magnetometer

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Abstract. This paper examines the relation between known crossings of the equatorward edge of the Earth's northern magnetic cusp during steady southward interplanetary magnetic field conditions, as recorded by the Magnetic Field Experiment on board the Polar spacecraft, and the predicted location of the equatorwardmost open field line as given by two differing magnetic field models. These are the Rice Field Model, an analytic open magnetosphere field model; and the Tsyganenko 1996 data-based model. The two models that were considered displayed remarkably similar performance in how often they can locate the dayside equatorward edge of the cusp in three dimensions. However, the Rice Field Model is superior for \( Z_{\text{eq}} > 5 R_E \), whereas the Tsyganenko 1996 model more accurately duplicates the observed range of cusp positions in the \( Y_{\text{geom}} \) direction. The Rice Field Model and the Tsyganenko 1996 models also differ in predicting the magnetic field vectors in and around the cusp region. The former is more accurate at high latitudes on closed dayside field lines equatorward of the cusp; the latter is better at predicting the field direction on interconnected field lines.

1. Introduction

In an effort to understand high-latitude magnetospheric phenomena such as the polar cusps, it is important to have magnetic field models that are reliable in this region. This paper compares the theoretical position of the equatorwardmost open field line from two different terrestrial magnetic field models for given times along the orbital path of the Polar spacecraft to the actual location inferred from the Magnetic Field Experiment (MFE) [Russell et al., 1995] observations onboard that satellite.

1.1. Observations

Polar is a satellite in a highly eccentric near-polar orbit, with a perilune altitude of 1000 km over the South Pole, and an apoapsis, geocentric distance of \( 8 R_E \) over the North Pole. It is designed primarily to observe the high-latitude magnetosphere with several instruments. The MFE consists of a pair of magnetometer sensors located on a 6-meter boom outbound from the Polar satellite main body, plus the associated electronics within the spacecraft structure. When the satellite exits the cusp, the MFE instrument observes a diamagnetic depression in the strength of the observed magnetic field [Zhao et al., 1997]. This is due to the magnetization currents associated with the particles in the cusp. Owing to the dynamics of the Polar spacecraft orbit, the cusp is carried from equatorward to poleward during the Northern Hemisphere winter, and from poleward to equatorward in the fall. Near the inflection the orbit roughly follows the terminator.

1.2. Previous Work

Several recent studies using data from both the Jovian and Saturnian satellites have attempted to define the configuration of the polar cusp and the near-cusp magnetopause. Zhou and Russell [1997] used the location of the bifurcation in the magnetic field to define the location of the polar cusp and found that the magnetopause radius increased suddenly tailward of the cusp. Zhou et al. [1999] found that the invariant latitude of the polar cusp varied with the tilt angle as it had been found to do at lower altitudes. Additionally, they noted that the average location of the cusp agreed more closely with the earlier empirical model of Tsyganenko [1989] than the more recent model [Tsyganenko, 1995, 1996]. Zhou et al. [2000] and Tsyganenko and Russell [1999] noted that the magnetic field around the cusp is not accurately modeled. The present empirical models overpredict the strength of the magnetic field equatorward of the cusp and underestimate it behind the cusp. Zhou et al. [2000] also show that the north-south and east-west components of the interplanetary magnetic field move the cusp location in latitude and longitude. Last, Forsyth et al. [2000] and Russell and Forsyth [2000] have shown that different models of how the cusp responds to pressure can result in very significant changes in the interpretation of the amount of physical properties acting at the magnetopause. Hence it is very important to develop accurate models of the polar cusp as a function of tilt and solar wind conditions.

1.3. Magnetic Field Models

The precession of the Rice Field Model (RFM) model is the Tsyganenko-Heel 1950 (THS) field model, which is a purely analytic field model based on the Voigt closed magnetosphere model [Voigt, 1972, 1981], perturbed to allow reconnection between interplanetary and terrestrial field lines. The model contains representation of the Earth's dipole and magnetoelliptical current systems and their associated magnetopause structure.
currents, and an interconnection field calculated from flux conservation for a given assumption about the location and strength of dayside merging. The boundary (magnetopause) zone of the magnetosphere in T1993 is a hemisphere with a cylinder attached to the antisunward side. The radius of both is fixed at 20 Rp. The location of Earth along the axis of symmetry moves in accordance with the prevailing solar wind conditions so that pressure balance at the nose is maintained (Toffolato and Hill, 1989, 1993). The IMF is a refinement of T1993, which includes a ring current and a variable diameter of the hemispheric-cylinder magnetopause shape (Zong et al., 1996). The version used in this survey is further modified to take Dst into an input to determine the strength of the ring current in the model for the given times of observed cusp crossings. Both of these models postulate merging of solar and terrestrial field lines along a line on the magnetopause crossing the subsolar point at an angle from the equatorial plane that is half the clock angle of the interplanetary magnetic field (IMF). The normal component decreases rapidly with increasing distance from this X-line, and the merging rate is maximum at the subsolar point for all IMF conditions (Toffolato and Hill, 1993), contrary to the claim by Newell et al. (1995) that T1993 and therefore the IMF has predominantly high latitude reconnection. The field line plots presented in Figures 1a and 1b illustrate this. Figure 1a shows field lines plotted for the RFM model and reveals that the nourwardmost open field line crosses the magnetopause very near Zcache = 0. A similar plot for the same time period (Figure 1b) using the Tyaganenko 1996 model shows the equatorward-most open field line (with respect to its location traced to the surface of the Earth) crossing the magnetopause at a high (southern) latitude. The Tyaganenko 1996 model (Tsy96) is a parameterized model based on actual observations of the field vectors. A statistical average of magnetic field vectors recorded by various spacecraft is used to constrain the model field instead of the purely analytic solution of the T1993 and the RFM models. Another major difference is the use of an elliptical revolution for the magnetopause shape (Tyaganenko, 1995, 1996). This hemiellipsoid eventually joins with a cylinder as well, approximately 70 Rp downstream from the characteristic point in space 5.48 Rp sunward of the Earth (i.e., approximately 64.5 Rp behind the Earth). The T1993 model and the RFM use a theoretical pressure balance formula (using solar wind dynamic pressure and IMF magnetic field pressure balanced against the pressure for all relevant fields within the magnetosphere) for the standoff distance (the distance between the center of the Earth and the magnetopause subsolar point). Tsy96 uses an empirically derived formula for standoff distance, which uses only the solar wind dynamic pressure as a variable. The reader is encouraged to investigate the source code for both of these models to understand the need arise to compare the standoff distance routines in more depth, as they are too lengthy to easily include herein. Tsy96 also eliminates all the J components of the IMF, not considering it as input or even keeping it for output, as Figure 1b shows via the purely vertical solar wind field lines when projected into the Z- Xmode plane.

Figure 1. (a) Field line plot for RFM for April 21, 1997 (this day's second cusp crossing). One field line is plotted for every 10 min of satellite travel time. Note merging of solar and terrestrial field lines near the Xmode axis, indicating low-latitude subsolar point merging, and contrast this with the latitude of the first open field line in Figure 1b. Solar wind conditions for the time near the cusp crossing are |Bz| > 300 km/s, solar wind speed, 400 km/s, density, 10 ion/cm³. (b) The same as Figure 1a, except for Tsy96 field lines. Vertical IMF lines are due to the lack of IMF Bz input into the Tsy96 code. Compare IMF field lines to those in Figure 1a, which have their Bz component, and are heavily draped over the magnetopause due to this Bz factor.
2. Procedure and Results

Using a list of Polar spacecraft cusp crossings provided by analysis of MFE data (X.-W. Zhou and C. T. Russell, http://www.giss.nasa.gov/polar/cusp/inf_list.html), all 141 crossings between April 1996 and May 1997 for which the Polar orbital plane was within 30° of the X-Gauss plane (i.e., spring and fall periods) were examined. The solar wind conditions for these data were then acquired from the Wind spacecraft averaged over 1-min and propagated to the subsolar point via the electromagnetic momentum vector (Elphic et al., 1989).

This electromagnetic propagation method gives the highest cross-correlation of the IMF dB vector during comparisons between observations from the Wind and IMP 8 satellites (Table 1). However, differences are not significant between this and a “ballistic” propagation method, using only the measured solar wind speed. It is, however, significantly better than a “simple” propagation which assumes a constant solar wind speed (which is equivalent to introducing a constant propagation time delay). The correlation coefficients between Wind and IMP 8 IMF data using several propagators are displayed in Table 1. More than 5000 data points were used to obtain these cross correlations under the conditions that both satellites be within 45° of the Earth-Sun line, and the solar wind flow be both supersonic and antisunward. These conditions ensure that both spacecraft are actually in the solar wind, and not the magnetosphere or magnetosheath (Gosling, 2000).

Wanting to find periods where the cusp would be expected to be in a fairly stable position, the 141 crossings were filtered for relatively steady (less than 40% deviation from the Bz value at time of MFE crossing) southward IMF after the propagation. If such steady periods were not selected, J would be ambiguous as to whether the satellite moved into the cusp or the cusp moved across the position of the satellite.

2.1. Dynamic IMF Input

Both models were run for the 35 orbit periods above the critical cutoffs, using a fully dynamic solar wind input in 1-min steps over the course of two hours (plus and minus one hour each side of the MFE crossing time). Field line traces in one-minute intervals along the Polar trajectory were then produced which were used to

Table 1: Cross-Correlation Coefficients Between Wind and IMP 8 Observations of Solar Wind Properties

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<th>Poynting</th>
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Table 1. Cross-Correlation Coefficients Between Wind and IMP 8 Observations of Solar Wind Properties

- Null
- Simple
- Poynting
- Ballistic

Evaluating which field lines were interconnected and which were closed along the satellite orbit. Although the models do not include polar cusp per se, the predicted equatorward edge of the cusp precipitation within the model can be obtained from the location of the first open field line (FOFL) [Reiff et al., 1977]. The FOFL is defined as the equatorwardmost interconnected field line given by the model along the satellite trajectory. Note that this corresponds to the open field line latest in spacecraft orbital time during the fall crossings. If the spacecraft passed from closed field lines directly to field lines which extend far down the tail, there should be no downward connecting solar wind particles on these field lines to cause the diamagnetic depression observed by the MFE instrument. Instances where the models showed this configuration were not considered to have located a FOFL. The time associated with the FOFL was compared with the IMF conditions to see if they still fell within the steady IMF range associated with the crossing. In the many instances where this was not the case, it was considered that the model did not find the cusp, since open field lines resulting from radially different IMF conditions cannot be fairly included in the statistics. The resolution that Bz did not change through zero between the predicted and actual crossing times was imposed as well. Both models performed almost identically, which is surprising, given their differing natures. Both are about the same in their rate of failure to find dayside interconnected field lines for a given orbit. Out of the 35 runs, Ty96 either returns no dayside interconnected field lines, or places them during times of IMF conditions not meeting the relevant criteria, 18 times (49%), and the RFM 15 times (43%). The GSM latitude (0°) of the FOFL, the angle between the X-Gauss (ecliptic) plane and the position where the equatorwardmost open field line was measured, is given by

$$\Theta = \tan^{-1} \left( \frac{Z}{\sqrt{X^2 + Y^2}} \right)$$

(1)

where X, Y, and Z are the GSM positional coordinates of the spacecraft at the time of the cusp crossing. Equation (1) is also used to produce the mapping of the FOFL by each of the models. Cross correlation coefficients (using only the orbits that fit the above criteria) of the model to be the GOE found by MFE for the RFM and Ty96 are 0.96 and 0.97 respectively. Standard deviation (σ) is 2.9° for the RFM and 2.3° for Ty96. Best linear fits yield

$$\Theta_{RFM} = 1.020 + 0.04$$

(2)

$$\Theta_{Ty96} = 0.920 + 0.40$$

(3)

The correlations and fits used herein are more for intermodel comparison purposes, and should not be taken literally in view of the fact that nearly half of the runs produced no dayside interconnected field lines. Figures 2a and 2b demonstrate how the FOFL found by the models compares to the observed GOE. Figure 3 takes a viewpoint looking down on the equatorial plane from above the Earth’s northern pole. This reveals that the FOFL positions successfully located by the RFM (represented as squares in the Figure 3) are located in the T-caused direction range between 1° and 1.5°, which is short of the observed range, particularly in the positive direction. This could be corrected by allowing a wider distribution of the T-caused field potential within the model. Successful Ty96 predictions of the FOFL (detected by triangles) cover the extent of the cusp position variations in Ty96 quite well, but with fewer anomalies at high latitude.
2.2. Constant IMF Input and Model Refinements

Using field models that changed configuration with each minute time step of the input IMF conditions, the POFL position of both models was often predicted just after a sharp change in the IMF near the time of the observed cusp crossing. To eliminate this effect, a second group of runs using a steady input, given by the average IMF conditions prevailing during the crossing were executed to see how the models acted without these dynamic disturbances. This also eliminated the need to filter the crossings again after the models were run, providing a cleaner comparison. Since the cusp re-enters within a few minutes to changes in the IMF, it is reasonable to assume the cusp has settled into a quasi-stable position if the solar wind conditions are steady for more than 5 min. Thus, a second group of model runs was performed using steady solar wind input given by the average solar wind conditions prevailing during the time period within 5 min before the observed cusp crossing. Changes of more than 10% or 20% in $B_y$, whichever was greater, were not allowed within this 5-min period. $B_y$ was limited to the greater of a 40% or 1-sigma change, with the additional assumption that the cases used has a significant change in either particle density or velocity. There were 24 cusp crossings conforming to these criteria.

Discrepancies in the predicted standoff distance between the two models by as much as $3R_E$ led to an investigation as to the cause. In comparing this value with an empirical fit from a statistical survey of magnetopause crossings using SEE 1 and 2 data [Shue et al., 1997] and later improvements [Shue et al., 1999], it was found that neither model (in the forms initially used in this survey) matched this empirical fit particularly well. The Shue et al. (1998) formula (their equation 10) is a function of IMF $B_y$ multiplied by a power of the solar wind dynamic pressure.

The standoff distance is $r_s$ (in $R_E$) and $B_y$ is the solar wind dynamic pressure in nano-Pascals. The dynamic pressure in this empirical formula was adjusted by assuming an alpha particle density of 4% that of the solar wind protons and traveling at the same velocity. In other words, the solar wind pressure was made 1.16 times greater than if one assumes only protons.

Ty96 uses an empirical formula and a characteristic distance (the same 4.5 $R_E$ $R_y$ mentioned previously) in lieu of a detailed pressure balance argument to find the standoff distance. However, Ty96 does not account for IMF magnetic pressure in any way to find the standoff distance, using only the dynamic pressure to calculate this. It was also found that Ty96 should be used with an additional 16% ram pressure from alpha particles (N. A. Tsyganenko, personal communication, 2000). In addition, Ty96 fails to account for the hydrodynamic correction factor for flow over a blunt object, whose value is usually given as 0.88 [Spritzer et al., 1965], which was erroneously included in the previous run. Disregarding this value and adding the additional alpha particle pressure is equivalent to increasing the solar wind dynamic pressure by 30%. Including this additional pressure improves Ty96's comparison to Shue et al.'s empirical fit significantly (changes from 0.3 to 2 $R_E$ have been seen). Since Ty97 uses dynamic pressure (instead of particle velocity and density) as an input parameter, the documentation does not make clear the need to make this 30% pressure adjustment, it is easy for a user to overestimate the standoff distance.

The IMF uses a purely theoretical pressure balance argument to find the standoff distance, and attempts to include as many fields as possible within the magnetic pressure term (both inside and outside the magnetopause). Values for some of these terms are
not well known. Others terms are not necessarily realistic, being artifacts of the model that must be included for completeness. The strength of the ring current magnetic field at the subsolar point of the magnetopause ($B_{rpm}$) is one of these unknowns. Though it could be determined once the strength of the ring current and standoff distance are known within the model, the fact that the standoff distance must be found before either of these can be calculated leaves only the option of estimating this value beforehand. An iterative process could be devised to more accurately find this value, but the increased accuracy (at most a few percent) over simpler schemes is unrewarding, taking into account the computational running time necessary to do this. In the original version of the RFM, the value of $B_{rpm}$ is set to a constant value. The RFM also includes a tail field intended to supplement the field lines in the tail, thereby relaxing the effects of the cross-tail current. Appendix A in the work of Tappinets and Haines (1989) describes this field, though the one used in RFM is even more simplistic. It is necessary to shield this field to prevent it from crossing the magnetopause, which results in a southward field component at the magnetospheric nose that must also be included in standoff calculations to achieve a consistent distance for the model, as it contributes to effects calculated later. Again, this value is not known in advance and is set to a constant in the original RFM code based on a working knowledge of what the model predicts. The RFM uses solar wind particle density and velocity as primary inputs to calculate ram pressure. The original code does not, however, take into account the effects of alpha particles.

A considerable improvement to the RFM standoff distance correlation with the Sun fit can be obtained by adding 4% alpha particles and letting the value $B_{rpm}$ be a function of IMF $B_z$. Since $B_{rpm}$ is a function of the ring current magnitude, it is therefore a function of $D_{st}$, which is a measure of the strength of that current. It has been shown that $D_{st}$ can be predicted by a function of IMF $B_z$ (Burton et al., 1975). Thus finding a best fit $B_{rpm}$ formula from the model $B_z$ input is justified. For the following runs $B_{rpm}$ was given a simple linear dependence on $B_z$ during southward IMF conditions and a constant value during northward times. To illustrate the difference the changes to the models have on southward distance, Figure 4 plots the RFM and Tsy96 in both their originally used and modified forms (as well as Shue et al.'s empirical fit). With these modifications, the Tsy96 standoff distance usually compares better with Shue et al.'s IMF $B_z > 3 \text{nT}$, but this is generally by a small amount (at most 0.5 $R_E$ if $B_z$ is southward). This can be seen in Figures 5 and 6. The RFM handles northward IMF with varying success, while Tsy96 is almost always correct during these times. When $B_z < 3 \text{nT}$, the improved system for the RFM generally performs markedly better than Tsy96, as Figures 4 and 6 illustrate. The RFM could easily incorporate the Shue et al. empirical formula for standoff distance. However, using this formula as a diagnostic to better refine the constants and variables within the RFM seems more elegant in that it preserves the analytic nature of the RFM. Furthermore, the Shue et al. formula, being empirical, does not necessarily reflect the instantaneous position. The modified RFM standoff distance lies within the statistical uncertainty of the Shue et al. fit for all southward IMF time periods studied in this paper. Incorporating the modifications listed above, both models were run for the 24 "constant IMF" condition case passes. Cross-correlation coefficients for the changed versions of the RFM and Tsy96 are 0.89 and 0.91, respectively. The standard deviation for the RFM is 4.2 $R_E$, and it is 3.9 $R_E$ for Tsy96. Linear fits for the steady
The reader should resist the temptation to compare the cross correlations (red and blue) of this section (equations (5) and (6)) to those in the previous section (equations (2) and (3)). Indeed, the lower cross-correlation coefficients in this section are a result of the fact that the models are not restricted to a smaller range of latitudes due to the relaxed selection rules allowed by having steady IMF input. In section 2.1, the farthest away from observation the model found the cusp, the more likely the IMF was to change. Since large changes in the IMF between the times of the observed cusp position and the model FOFL were not allowed, this led to a positive bias (for both models) in the correlations which the effects of this section attempt to eliminate.

Equations 7a and 7b show the latitudes of model 6 versus those observed by MPE. Though these fits and correlation coefficients are more realistic than those in section 2.1, these numbers still show a lack of a clear trend for the IMF field lines.

Viewing the cusp crossings from the perspective of the sun, the domain of the cusp is roughly a "Y" shaped in the X-Z plane. The cusp location in the Y direction is constrained at lower Zsolar values by the location of the generating dynamo near the center of the Earth, and has an increasing freedom of range in Zsolar with increasing Zsolar altitude. Figure 8 demonstrates that both models do a fair job of replicating the observed ranges of the FOFL at low Zsolar values. The IMF does not predict interconnected field lines if Zsolar is large, but does cover the Zsolar range well. Ty96 more accurately covers the range of observed cusp crossings below Zsolar = 5.5 R_E but fails to find any interconnected field lines above this. The IMF also exhibits a more characteristic than the values at lower Zsolar that the IMF does not have a wide enough distribution in local time of its interconnection magnetic field normal component, and Ty96 does not have a large enough normal component at high latitudes.

2.3. Field Vector as a Diagnostic of Model Performance

In addition to the position of the FOFL, the magnetic field vectors from the models and observations in the cusp region can be used to gauge model accuracy and verify the correctness of the IMF shape as well as its position. Difference comparisons between each model's field magnitudes and those observed, as well as the angles between the observed and model vectors formed by the projections of the vectors into the X-Z plane, were made. At this time, there are only four passes when both Ty96 and the IMF find interconnected field lines near (within 10-min spacecraft orbital time) the observed cusp

Figure 4. November 4, 1996. (a) Magnetospheric standoff distance variations due to solar wind dynamics. The solid line is the Shue et al. empirical form, the black dotted is the RFM refined standoff distance, and the dashed line is the Ty96 corrected standoff distance. The blue with plus and cross designators respectively show the RFM and Ty96 original, uncorrected standoff distances. The large disparity is largely resolved by the refinement applied to the models. This data was used only as a test for standoff calculations, as the IMF conditions are not dynamic for inclusion in the cusp locating runs. (b) IMF Bx (solid line) and By (dashed line) for the given time range as measured by Wind. (c) Solar wind pressure (solid line) and number density (dotted-dashed line). The solar wind velocity (Vsw) was steady between 400 and 460 km/s during this interval.
Figure 5. Same as Figure 4, but only showing the improved standoff distances for May 4 1997. $V_\infty = 420$ to 440 km/s. The vertical dotted line is where the RPM places the equatorward edge of the cusp, and the vertical dashed line is where Tay96 does so. This is an example of how both models like to place the first open field lines at nearby IMF features, even given 20 mile of steady IMF conditions prior to the MFE cusp time.

Figure 6. Same as Figure 4, but only showing only the improved standoff distances for October 27, 1996. $V_\infty = 390$ km/s, steadily increasing to 470 km/s. Note that the Tay96 standoff distance during strong southward IMF from 0.8 to 1.4 hours does not show erosion, but the RPM does. This data was not used for cusp crossing comparisons.
position, which is considered the criterion for a fair head to head comparison. Both the dynamic IMF and refined constant IMF runs were used in the selection of dates for this comparison. The selected dates were then run with the dynamic solar wind input and the versions of the models with the improved standoff distances. Three of these dates have very low latitude cusp positions and fair agreement between model and observed magnetic field

Figure 7. (a) Plot of latitude of the predicted FOFI from the IMF versus the observed latitude for the constant IMF runs using the refined standoff distances in section 2.2. The bold line represents a perfect fit, squares represent IMF data points, and the dotted line represents the best linear fit to the data ($\theta_{\text{ IMF}} = 0.87 \theta_{\text{ MFE}} + 6.6$). The time resolution of one minute in the data is roughly equivalent to 1.5 deg, making the size of the symbols in these plots approximately the size of the error bars. (b) The same as Figure 7a, except for Ty96 model runs shown as triangles. The dashed line represents the best linear fit to Ty96 data ($\theta_{\text{ IMF}} = 0.85 \theta_{\text{ MFE}} + 6.0$). On average, Ty96 places interconnected field at lower latitudes than IMF or MFE observations.

Figure 8. View of FOFI positions in the $F-Z_{\text{ IMF}}$ plane from the perspective of the Sun. Ty96 is unsuccessful in finding interconnected field lines above 3.5 Rs. The restricted range of the IMF in $Z_{\text{ IMF}}$ is illustrated by the shaded region in the left panel. Shading in the right panel emphasizes the limited $Z_{\text{ IMF}}$ range of the Ty96 model.
magnitudes. Figure 9a shows the difference between MFE and model magnitudes for one such pass. Closed dipole field lines extend from 0 to 1 hours (normalized time) on the plot, interconnected field lines with solar wind particles precipitating downward from 1.0 to 1.17 hours (shaded region), and tail (plasma mantle) field lines from 1.17 hours onward. This data (April 21, 1997) shows the most variance between observation and prediction of the three low-latitude corps. Try96 starts close to the observed magnitude (over the equator) and steadily worsens until about 6 minutes into the cusp. The RPF starts out overestimating the field strength, gradually moving through better predictions until finally underestimating it in the region poleward of the cusp. The maximum percentage difference between the predictions and observation is 12% for Try96 and 8% for the RPF. These three events also place Polar lower than 5 R_E altitude while traversing the northern portion of the cusp. This is consistent with Zhou et al. (1997), who find that MFE shows B_0 to be depressed least for low-latitude cusp. Trykowski and Russell (1999) show that below altitudes of 5 R_E, the increasing strength of the geomagnetic field overshadows the value of the magnetic field depression from demagnetization.

The higher-latitude (90°) and altitude (6.5 R_E) crossing of October 9, 1996 (Figure 10) shows more interesting variations between the observations and the models. Note that the event has interconnected field lines from 0.0 to 1.0 hours and closed field lines from 1.0 hours onward. Since these two events occurred 6 months apart, Polar was moving equatorward in this instance instead of poleward as it was in Figure 9. For the event depicted in Figure 10, both models show fair magnitude correlation with MFE data on closed dipole field lines (error between 0 and 15%), but this changes rapidly on interconnected field lines. Maximum error between the predictions and observations within the cusp is dramatically higher than the previous example, being of the order of 50% for both models. Zhou et al. (1997) pointed out that Try96 suffers at high latitudes near the cusp region and typically predicts too strong a field within the cusp. This is confirmed here and is true for the RPF as well.

The angle α is defined as the angle between the observed and model magnetic field vectors projected into the X-Z plane. The RPF is usually quite accurate in predicting the magnetic inclination (i.e., α = 0) on the closed field lines before the equatorward edge of the cusp. Try96 is more accurate than the RPF on open field lines (shaded regions of Figures 9 and 10), though it typically diverges in this region as well. Interestingly, Try96 is often quite accurate at predicting α = 0 in the immediate vicinity of the cusp crossing. The increasing values of α on interconnected field lines could be corrected in several ways within the RPF, modifying the tail field, increasing the magnetosheath flow speed, or changing the distribution of the normal component of B along the X-line.

The angle β is defined as the angle between the observed and model magnetic field vectors projected into the X-Y plane. Both models do approximately the same with β, being quite accurate on closed dipole field lines. In this region, both models diverge within 5 deg of the observed direction for all four events studied, with Try96 usually being more accurate. Within the region of interconnected field lines, variations of α as high as 20 deg are observed on two of the dates considered (see Figures 9 and 10). Both of these dates have strong IMF B_z components. The other two dates (not depicted) have weak IMF B_z components compared to B_Y and the models do equally well predicting β on open and closed field lines (less than a 5 deg variation...
Figure 10. The same as Figure 9, but for October 9, 1996. This crossing is 6 months earlier than the one depicted in Figure 9, so these panels should be considered in the opposite direction (i.e., the closed dayside field lines are after the cusp traversal at 1.0 hour instead of before). This crossing is also at higher latitude and altitude than the April 21 event. The shaded region represents the region of open field lines with magnetic disturbances as observed by MPE, extending over a much longer time interval than the previous example.

Throughout) Thus variations of $\beta$ away from zero on interconnected field lines may be due to the effects of magnetic tension not accounted for in the models.

It can be seen from Table 2 that both models follow the trends in the observed magnitude quite well, but have a large offset from the measured value. The RGM magnitude offset is less than that for Toy96 because the cusp in three of the four cases, and has a greater offset in three of the four cases poleward of the sunward edge of the cusp (on open field lines). Variations in the vector angles in the $X$-$Y$ plane ($A$) are also well predicted by both models on closed field lines, and worse on open field lines. The offsets for $A$ show that both models predict this angle fairly well on closed field lines. Though both diverge on interconnected field lines, the RGM offset is closer to observation in three out of four instances, though Toy96 is marginally better with the correlation coefficients and linear fit slope values in this region. Referring back to Figure 9 shows that despite the small difference in the values in Table 2, Toy96 is visibly better in a direct comparison of the data for this case. In only one case (for May 17, 1996) the models do well predicting the vector angle variations in the $X$-$Y$ plane ($B$). This data had a large IMF $B_z$ to $B_y$ ratio (particularly during the time Polar was crossing open field lines) so there was little magnetic tension in the $Y$ direction to direct the field lines. In the other three cases, both models are unconvincing predictions of this angle on open field lines. On closed field lines, both do poorly predicting $B$ half the time. Inspection of Figures 9 and 10 will serve as a reminder that correlation coefficients and linear fits are sometimes deceptive. Figures 9 and 10 show that both models seem to do well predicting the general behavior of the observed vector angles in the $X$-$Y$ plane on closed dayside field lines despite the very poor correlations and linear fits for those dates (October 9, 1996, and April 19, 1997).

In general, the RGM is more accurate at predicting high-latitude field vectors on closed dayside field lines, once away from the equatorial region where Toy96 benefits from the large number of data points available to it there. The RGM is also more accurate in predicting $A$ in this same region. Once within the cusp, Toy96 typically (but not always) becomes the more accurate of the two for both magnitude and $A$. The Toy96 superior ability to predict $A$ on open field lines is likely due to the benefit it receives from its data dependence. The Toy96 database includes some of the diamagnetic effects from precipitating particles even though it does not model this effect directly. The RGM has no mechanism to include this. Variations in the angle $\beta$ are almost indistinguishable between the models. The reader should keep in mind that, since there are only four events studied in detail at this time, these conclusions should not be considered statistically reliable until more events can be added to this portion of the study.

3. Discussion and Conclusions

The two models examined here exhibit similar performance in their ability to locate the sunward-most open field line in comparison to observation, placing most of the FOFL within a few degrees of the observed equatorward edge of the cusp magnetic cavity. However, in a significant number of cases, both also fail to locate any dayside interconnected field lines that can be associated with Earthward precipitating solar wind particles. This failure ~ 30 - 50% of the time, depending on whether one
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The magnetic field data are shown in the "decade" field lines, all of which are closed and counterward of the tip, and the "megamode" field lines, which are predominately interconnected with the solar wind and represent cusp or plasma mantle field lines. It is the correlation coefficient between observation and model prediction. One should use only to compare it values between similar data sets (e.g., Bm vs. Ty96 only the either decade or megamode field lines on a single day) due to the differing the values of B. The A and B values are the angle the field vectors (from observation as well as the models) make from the z-axis in the X-T96 and X- R96 planes respectively. The difference between A from observation and that of the model gives the value of a. It is also true for B. The values of m and n represent the slope and y-intercept (model offset from observation) of the linear fit between observation and model as {y} = {m} + {n}.

Consider the constant or dynamic IMF input runs. The RPM performs much better than Ty96 at higher altitudes (Zmax > 5.5), while Ty96 does a better job of predicting the spread of FOFl positions in the Rmax direction. The failure of the RPM to predict the range of FOFL positions in Xmax is thought to be due to the distribution of its interconnection field component normal to the magnetopause. Since the cross-polar cap potential (studied extensively in the work of Boyle et al. [1993]) determines the total normal component, but the distribution of its is unknown, the total amount is spread across the dayside using a cusp cosine function in the current version of the RPM. The construction of the RPM FOFL positions in Figures 3 and 4, as well as the analysis of field line vectors in the cusp region (see section 2.3), strongly suggests a wider distribution of the normal component is needed within the RPM to keep reconnection from being too concentrated at the nose. This narrow distribution can also account for the lack of high-latitude field line merging observed within the RPM, as there is no significant amount of reconnection far away from the subsolar point with the current scheme.

Preliminary examination of the field line shapes near the cusp region show that the models' high-latitude field configurations differ in a crucial systematic way. It is interesting that the RPM performs better on high-latitude closed dayside field lines, while Ty96 models open field lines somewhat better. The success of the RPM equatorward of the cusp may mean that Ty96 would be improved with a blunter nose shape. Ty96 does have a parameter (usually left unchecked) that can adjust the ellipticity of its magnetopause. Since the hemisphere is tilted to a cylinder -45 degrees down the tail, this will also change the diameter of the magnetopause. Figure 2 of Tappinowski [1998] suggests that this would systematically move the cusp equatorward in the model.

This aggravates the preexisting problem in which Ty96 places interconnected field lines at latitudes that are too low [Zhou and Russell, 1997]. This can also be seen in Figure 7b, which shows the major arc of the Ty96 predicted FOFL lines are below those observed. Allowing a change in the location of the hemispheric/field center may help to alleviate this problem. Ty96's somewhat better handling of interconnected field line direction vectors suggests a more accurate tail field configuration compared to what is used in the RPM. Swot are now being taken to integrate a more realistic tail field configuration into the model.

Both models lack a strong enough ratio of Z to X components of the magnetic field vectors high in the plasma sheet compared to observation (e.g., they are not vertical enough). See Figure 11a and 11b for a clarification on this point. If one includes a "dent" in the polar cusp region of the magnetosphere, the magnetospheric field lines of an analytic model become markedly more vertical in the cusp region, as Figure 12 shows [Kuocek and Toffolato, 1999]. Figure 12 plots data from the Kuocek and Toffolato "finite element" magnetosphere model in the region of interest and illustrates the impact of inclusion of a "dent" on high latitude field line orientation. The Kuocek and Toffolato model is not yet fully operational but could be interesting to include in a future study on the magnetospheric fields. Tappinowski and Russell [1999] investigated inclusion of such a deformation of the magnetopause in Ty96. Their Plates 1 and 2 also reveal more vertical field lines in the cusp region as a result of this feature. It should be noted that Zhou and Russell [1997] find little evidence for this suspected indentation, so there is good reason to spend more effort improving the existing models.

Future efforts will be aimed at attempting to improve the RPM. Comparison to high-latitude data from the THEMIS spacecraft, particularly from the MFE instrument, will be very useful in
helping pin down some of the less well known quantities in the
analytic model. Once the direction of the field line vectors within
the cusp are improved, the differences in magnitude between
the model and observation can be used to infer the particle fluxes
within the cusp. These particle fluxes and the associated
magnetization currents are presumably the reason the observed
field magnitudes are so diminished from theoretical values. While
Tsy96 may be more accurate in that it does evidently include
some of the effects of magnetization currents, the complete lack
of these within the RFM might be used to good advantage. The
difference between the RFM field magnitudes on interconnected
lines and those observed can be directly related to the strength of
the magnetization currents. Since it is unknown to what degree
Tsy96 resolves magnetization currents within its database, the
actual value of these currents is not easily obtained from a model
to observation comparison. The magnetization currents inferred
using the RFM can be compared to plasma data from the TIMAS
or HYDRA instruments also aboard the Polar satellite as a further
diagnostic for the model. It may be possible to find a correlation
with IMF parameters that would allow estimations of these
magnetization currents to be directly included within the RFM.
Both the RFM and Tsy96 exhibit the correct qualitative
dependence on solar wind and IMF parameters. However, neither
model can provide in its present form a quantitatively correct
mapping of dayside field lines. Since traces of field lines starting
or ending at high latitudes are often used to study phenomena on
or near the equatorial plane, it is important to improve these
models’ high-latitude accuracy for the benefit of all fields of
terrestrial space physics. The conclusions of such work may be
strongly affected by the realization that model field line vectors at
high latitudes are not particularly accurate. MFE observations
provide a powerful diagnostic for testing the accuracy of the field
models and reveal ways to improve them. Further studies using
MFE data, as well as other instruments aboard the Polar and

Figure 11. (a) Field line plot of the RFM field lines for October 9, 1996, with model (outlined) and MFE (bold black)
veRs over-plotted for comparison. The vertically oriented arc segment is the Polar trajectory. The FPI is
in the middle and +45 minutes of field lines in 15-min spacing are shown on each side. Closed field lines are
denoted by crosses and open (interconnected) lines by open circles. No growing error in the RFM vector direction
(a single shown) and magnitude once within the cusp region. (b) The same as Figure 11a, except that Tsy96 field
lines and vectors are plotted with MFE vectors. Tsy96 is more accurate in vector direction on open field lines but
does not better with the magnitude in this region.

Figure 12. Plot of data from the Kloeck and TufoIetto finite
element model [1999], demonstrating the increased sweep back
of field lines due to a magnetospheric dent above the polar cusp.
Solid lines are field lines for a symmetric magnetopause, while
dotted lines are for a boundary with the “dent” in the cusp region
of the magnetopause.