The determination of shock ramp width using the noncoplanar magnetic field component

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Abstract. We determine a simple expression for the ramp width of a collisionless fast shock, based upon the relation-
ship between the noncoplanar and main magnetic field com-
ponents. By comparing this predicted width with that mea-
sured during a shock observation, the shock velocity can be
determined from a single spacecraft. For a range of low-
Mach, low-$\beta$ shock observations made by SSEE-1 and-
-2, ramp widths determined from two-spacecraft compari-
son and from this noncoplanar component relationship agree
within 30%. When two-spacecraft measurements are not
available or are inefficient, this technique provides a reason-
able scale size estimation for low Mach shocks.

1. Introduction
The determination of spatial scales within the collision-
less shock front is a central problem of observational shock
physics. Of particular interest is the width of the shock ramp,
defined as the main transition layer between upstream and
downstream plasmas. However, without spacecraft measure-
ments in a spatial frame of reference, it is impossible to make
comparisons between observations and theoretical models.

For bow shock studies, generally one of two methods is
applied to transform the time series observed by an in-situ
magnetometer into a spatial profile: (1) the comparison of
shock observations made by multiple spacecraft with known
separations in time and space (e.g., Russell et al., 1992;
Farris et al., 1995; Newbury and Russel, 1996), and (2)
the comparison of the duration of the shock foot with the
foot length predicted by a model based on the motion of
specularly reflected ions (e.g., Schepke et al., 1983; Gosling

Both methods assume that the bow shock is stationary and one-
dimensional, and each has its own limitations. The first is
not reliable when the time delay between spacecraft obser-
vations is too large (non-stationarity may affect the results) or
too small (relative errors become large). Also, large trans-
verse spacecraft separations with respect to the shock front
can introduce error due to the three-dimensional nature of the
bow shock. The second method cannot be applied to
laminar shocks (shocks observed during low Mach and low
$\beta$ conditions). At such shocks, ion reflection does not play a
dominant dissipative role, and no foot structure is observed.

Because of these limitations, it is desirable to have another
independent method for measuring shock scale lengths, par-
ticularly for laminar shock observations made by a single
spacecraft. In this paper, we make use of the noncoplanar
component of the magnetic field within the shock ramp in
order to estimate a scale size. This scale size is then com-
pared with a variety of low Mach number shock observations
made by the SSEE-1 and -2 spacecraft.

2. Theoretical Basis

Within the ramp layer of a fast collisionless shock such as
the Earth’s bow shock, the magnetic field is observed to
rotate out of the coplanarity plane (the plane defined by
the shock normal and the upstream and downstream magnetic field vectors) [Thomsen et al., 1987]. The analytical rela-
tion between this noncoplanar component and the main mag-
netic component of the shock profile was first derived pne-
meteorologically by Jones and Ellison [1987] in an integral form; its approximate nature has been shown observation-
ally [Gosling et al., 1988; Friedman et al., 1990]. Recently,
Gedalin [1996a] examined the noncoplanar component us-
ging a general two-fluid hydrodynamics approach, and carried
out the derivation with only the widely accepted assump-
tions of shock stationarity, one-dimensionality, and quasi-
neutrality. In the coordinate system where $N$ denotes the di-
rection along the shock normal, $L$ is transverse to the shock
plane, and $M$ is directed out of the plane, the general expres-
sion for the noncoplanar magnetic field component ($B_M$) is:

\[
B_M (1 - \frac{B_N}{4\pi ne^2}) = \frac{c B_N}{4\pi ne^2} dN \cdot dB_L - \frac{c \cdot d}{ne^2} \left( p_{N}^{(1)} + B_N (p_{N}^{(1)} + p_{N}^{(2)}) \right) \left( \frac{\partial N}{\partial N} \right) \nonumber
\]

in the limit, $n_e \rightarrow 0$, and where $N$ is the $N$ component of the
hydrodynamic velocity; $B_N = \text{const}$ and $n_e = \text{const}$; and $\n$, $P_N$ are components of the pressure tensor.

It has been shown by Gedalin and Zilberter [1995] that
the main contribution to $P_{NM}$ is primarily due to the pres-
ence of strong ion reflection and the consequent ion gyra-
tion. For low Mach number shocks, significant ion reflec-
ction is neither expected or observed, and $P_{NM} \ll n_e m_i V_i^2$.
[Gedalin, 1996]. The P_{\text{inj}} term is related to the anisotropy of the electron pressure, which is not typically large, especially in a low-$\beta$ plasma. Dropping terms, (2) reduces:

$$\tilde{\beta}_{M} = \frac{I_{w}}{1 - \left(\frac{\cos \theta_{\text{inj}}}{M_{A}}\right)^{2}} \frac{dN}{dB_{z}}$$

where $I_{w} = c \cos k_{x} \left(\frac{M_{A} \omega_{i}}{\omega_{p}}\right)$ (i.e., $k_{z} = 1/I_{w}$ is the wavenumber of a whistler, phase-standing upstream of the ramp), $\theta_{\text{inj}}$ is the angle between the upstream magnetic field and shock normal, $M_{A}$ is the Alfvénic Mach number, and $c/\omega_{i}$ is the ion inertial length. $(\cos \theta_{\text{inj}}/M_{A})^{2}$ is usually small for quasi-perpendicular shocks, but is included for completeness. Note that (2) is a differential analog of the integral relation developed by Jones and Ellison [1997].

By measuring the slope of the main magnetic field component ($dB_{z}/dt$) within a shock ramp observation and relating it to the local noncollinear ramp component according to (2), one can determine an independent estimate of the velocity of the shock front. Once the shock velocity is determined, it is then elementary to transform the observed temporal shock profile into a spatial one suitable for comparison with theory, other shock observations, or simulations.

For the observationist, further difficulties can arise from (2) since it is very sensitive to gradients in the field profile; noise and wave activity associated with a typical bow shock observation can make localized measurements of $dB_{z}/dt$ difficult. Traditional filtering does not preserve gradients well and can obscure the width of the shock ramp. For laminar shocks, we may apply a simple model which approximates the shock ramp with a hyperbolic tangent:

$$B_{z} = \frac{B_{w} + B_{c}}{2} + \frac{B_{c} - B_{w}}{2} \tanh \frac{3N}{I_{w}}$$

where $B_{w}$ and $B_{c}$ refer to the L-component of the field upstream and downstream of the shock front, and the coefficient 3 ensures that 90% of the magnetic field variation occurs within the ramp (defined in the region $-I_{w}/2 < x < I_{w}/2$). The point where $B'_{M}$ is a maximum is easily identified and will theoretically be located in the middle of the ramp (where $N = 0$). Applying (2) to (3) for $B'_{M,\text{max}}$, one has a simpler expression for the ramp width:

$$I_{w} = \frac{3}{2} \left(\frac{B_{c} - B_{w}}{B_{M,\text{max}}^{2}}\right)$$

This approach requires accurate measurements of $B_{w}$ and $B_{c}$, but is less sensitive to the local $B_{z}$ gradient than the direct application of (2) to an observed shock profile.

3. A Sample Bow Shock

In the present section, we apply the proposed method to a quasi-perpendicular collisionless bow shock crossing that was observed by the ISEE-1 and -2 spacecraft on Nov 26, 1977 at 06:10 UT. Data from the fluxgate magnetometers is filtered to obey the Nyquist criterion and then sampled at the rate of 16 vectors/sec. By averaging over a minute of data upstream and downstream of the shock front and applying the coplanarity theorem, the shock normal is determined, and $\theta_{\text{inj}}$ is found to be 67°. Figure 1 shows the high-resolution ISEE-1 observation of the magnetic field, rotated into the coplanar frame.

Plasma measurements of the upstream solar wind are obtained by the ISEE-1 and ISEE-3 solar wind experiments, and are used to calculate the following parameters: $c/\omega_{p}$ is 58 km/s; $M_{A} = 2.7$ (so that $I_{w} = 8.3$ km/s); and electron and ion beta, $\beta_{e} = 0.36$ and $\beta_{i} = 0.16$.

In order to reduce short wavelength noise while maintaining the gradients within the shock profile, the data was smoothed by applying a discrete wavelet transform (using the Daubechies-10 wavelet) and removing the 6 finest scales [e.g., Chau, 1992; Donoho, 1993]. This corresponds to the removal of features whose scales are shorter than 64 data points (= 2^6), which in turn corresponds to ~4 sec averaging for this high resolution data. Although substantial oscillations persist in the upstream and downstream regions, the behavior of $B_{M}$ and $B_{z}$ within the ramp is consistent with the theoretical prediction, as seen in Figure 2. Comparison of $B'_{M,\text{max}}$ with the slope of $B_{z}$ according to (2) results in a shock velocity estimate of $V_{sh} = 4.4$ km/s. Independently, the shock velocity calculated from the ISEE spacecraft sep-
Figure 3. Agreement between the ramp measurement tech-
niques vs. the deviation of $B_{92}$ during the ramp observation. When $B_{92}$ remains constant, the noncoplanar technique is most accurate. Open circles indicate supercritical shocks.

ation is $V_{sa} = 5.7$ km/s (with a separation $L_{x} = 20$ km along the shock normal and ramp crossing time separation of 3.5 s). The two estimates agree within 25% deviation. Applying the tanh approximation from equation (3), the ramp width is estimated to be $47.7 \pm 7.5$ km. Based upon two-spacecraft comparisons, ramp width is found to be $56.7 \pm 8.2$ km. (The temporal duration of the ISEE-1 ramp obser-
vation is approximately 10.25 s) These two estimations of ramp width agree within 20%, which is considered very sat-
isfactory. The error in both calculations is primarily domi-
nated by the uncertainty of the shock normal direction (which affects measurements of the shock velocity, $\theta_{PSV}$, the in-
cluded solar wind flow, etc.). The normal is determined via the coplanarity assumption: deviations in the measure-
ments of the average upstream and downstream fields propa-
gate through the coplanarity calculations, and are significant (even after the wavelet filtering of the noise on the profile).

4. Application to a Variety of Shocks

In order to estimate the reliability of the method out-
lined in the previous section, here we compare the results of the proposed approach when applied to a variety of low-
Mach shock observations. Table 1 contains relevant param-
eters for a selection of shocks observed by the ISEE space-
craft: the Alfvén Mach number ($M_{A}$), ratio of criticality ($R_{e} \equiv M_{AS}(M_{e})$, where $M_{AS}$ is the magnetosonic Mach number and $M_{e}$ is the critical Mach number), $\theta_{PSV}$ (as de-
termined by coplanarity), total $\beta$ of the upstream plasma, and measurements of ramp width using the two spacecraft ($t_{r,AS}$) and based on $B_{92}$ (median, $t_{r,92}$). These shocks were se-
lected for their low-$\beta_{92}$, Mach, quasi-perpendicular char-
acteristics; in addition, these shocks were observed at times when the ISEE spacecraft configurations were ideal for de-
termining finite spatial scales (i.e., small spatial and temporal separations between observations, and $\theta_{PSV}$ calculated by coplanarity and via an ellipsoidal bow shock model across within 10%). Nearly-perpendicular shocks are avoided due to the large errors associated with determining shock normal vectors when $\theta_{PSV} > 80^{\circ}$. (Also, perpendicular shocks may not have the same whistler mode structure as shocks with lower $\theta_{PSV}$ [Newbury and Russell, 1996; Friedman et al., 1990].) Many of these shocks have been examined previ-
ously by Farriss et al. [1993].

In Table 1, the ratio of the ramp measurements indicates how well the estimations agree. For eight of the ten shocks in Table 1, agreement between the two techniques is satis-
factory (within 30%). Also, they are comparable even when the shock is no longer strictly laminar: several of the shocks listed in Table 1 are slightly supercritical ($E_{92} > 1$) and are associated with a $\beta$ that isn’t especially low ($\beta > 0.3$). The shocks observed on 79 Aug 13 and 79 Nov 26 are exceptions: equation (3) does not accurately estimate their ramp widths. The 79 Nov 26 shock is clearly supercriti-
cal, and despite the presence of an identifiable non-coplanar component in its ramp, it is expected that ion reflection is a dominant processes at such a shock. The pressure terms in (1) are not always small, and (2) is not applicable.

The 79 Aug 13 shock is only slightly supercritical, and similar shocks on 77 Nov 26 and 78 Jan 6 agree quite well with (4). This discrepancy can be explained by considering the effects of turbulence and two-dimensional disturbances within the shock profile, as evidenced in the deviations of the $B_{92}$ component within the ramp layer. Equation (2) as-
sumes that $B_{92}$ remains constant throughout the shock obser-
vation, but in reality this is not always so. Two-dimensional disturbances and plasma turbulence on the shock front can obscure the coplanarity rotation. In Table 1, the columns la-
belled $(\Delta B_{92})/B_{92,max}$ lists the maximum deviation of the $B_{92}$ component during the shock ramp observation, normal-
ized to the $B_{92,max}$ in the ramp. Within the shock ramp on 79 Aug 13, fluctuations of $B_{92}$ are on the order of $B_{92,max}$.

<table>
<thead>
<tr>
<th>Date, Time [UT]</th>
<th>$M_{A}$</th>
<th>$R_{e}$</th>
<th>$\theta_{PSV}$</th>
<th>$\beta$</th>
<th>$t_{r,AS}$ [km]</th>
<th>$t_{r,92}$ [km]</th>
<th>$t_{r,20}$ [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>77 Nov 26, 0610</td>
<td>2.73</td>
<td>1.16</td>
<td>67.0</td>
<td>0.52</td>
<td>0.28</td>
<td>47.7 \pm 7.5</td>
<td>56.7 \pm 8.2</td>
</tr>
<tr>
<td>77 Nov 26, 0619</td>
<td>3.07</td>
<td>1.32</td>
<td>69.2</td>
<td>0.62</td>
<td>0.15</td>
<td>52.3 \pm 9.1</td>
<td>50.0 \pm 8.6</td>
</tr>
<tr>
<td>78 Jan 06, 0701</td>
<td>3.21</td>
<td>1.35</td>
<td>58.4</td>
<td>0.25</td>
<td>0.13</td>
<td>73.6 \pm 9.8</td>
<td>88.4 \pm 1.5</td>
</tr>
<tr>
<td>78 Aug 27, 2007</td>
<td>2.23</td>
<td>0.85</td>
<td>74.6</td>
<td>0.16</td>
<td>0.38</td>
<td>127.0 \pm 1.2</td>
<td>100.3 \pm 5.6</td>
</tr>
<tr>
<td>78 Aug 28, 0009</td>
<td>1.65</td>
<td>0.67</td>
<td>53.5</td>
<td>0.05</td>
<td>0.26</td>
<td>166.0 \pm 1.5</td>
<td>97.6 \pm 0.7</td>
</tr>
<tr>
<td>79 Aug 13, 1427</td>
<td>3.75</td>
<td>1.40</td>
<td>78.7</td>
<td>0.10</td>
<td>0.83</td>
<td>34.4 \pm 1.1</td>
<td>66.7 \pm 2.4</td>
</tr>
<tr>
<td>79 Sep 18, 1029</td>
<td>2.92</td>
<td>1.15</td>
<td>62.3</td>
<td>0.18</td>
<td>0.03</td>
<td>92.2 \pm 1.3</td>
<td>83.2 \pm 1.5</td>
</tr>
<tr>
<td>79 Nov 26, 0015</td>
<td>6.00</td>
<td>2.69</td>
<td>67.5</td>
<td>0.83</td>
<td>0.71</td>
<td>271.0 \pm 4.6</td>
<td>154.0 \pm 5.9</td>
</tr>
<tr>
<td>80 Sep 06, 0106</td>
<td>2.44</td>
<td>0.98</td>
<td>61.6</td>
<td>0.17</td>
<td>0.34</td>
<td>75.5 \pm 9.3</td>
<td>90.0 \pm 6.6</td>
</tr>
<tr>
<td>80 Dec 19, 1435</td>
<td>1.67</td>
<td>0.62</td>
<td>74.8</td>
<td>0.04</td>
<td>0.07</td>
<td>102.0 \pm 5.5</td>
<td>100.0 \pm 5.8</td>
</tr>
</tbody>
</table>
resulting in an under-estimated ramp width from (3). Even with the stringent requirements placed on the selection of shocks in Table 1, non-stationarity and turbulence are still a fact which cannot always be ignored. In Figure 3, the ratio of the two ramp measurement techniques are compared with the deviation of $B_{x}$ during the ramp observation (and normalized to $B_{x,max}$). The shock ramps where the two techniques agree best also have the most constant $B_{x}$. However, even a noticeable deviation in $B_{x}$ can still result in a reasonable estimation of ramp width (for example, the 38% deviation of $B_{x}$ with respect to $B_{x,max}$ for the 78 Aug 27 shock). The open circles in Figure 3 correspond to supercritical shocks.

For shocks where a foot structure could be discerned, we applied the foot measurement technique outlined by Gosling and Tomsmen [1958] (Table 2), and compared it to the foot length measured by the two spacecraft ($L_{foot}$). For the most supercritical shock (79 Nov 26), the foot measurement technique works well and the non-collinear measurement fails. It is interesting to note that both scale measurement techniques fail for the 79 Aug 13 shock. The remaining observable shock feet agree well with the prediction, generally with uncertainty comparable to that of the non-collinear technique. Our case (60 S) 66 has a large degree of uncertainty (50%), which is primarily due to the difficulty of identifying precisely where the shock front begins and ends.

5. Conclusions

We have examined the relationship between the non-collinear component and gradient of the main magnetic field component within the collisionless shock ramp. By estimating the scale size of the ramp width based upon this relationship and comparing that length with the temporal duration of a shock ramp observation, we calculate the shock velocity in the spacecraft frame. This enables the observed shock profile to be transformed into a spatial frame, suitable for comparison with other shock observations and with theory. Assuming that turbulence and any two-dimensional disturbances to the shock front are kept to a minimum, equa-

tion (2) should be valid for low-Mach number shocks since the most "dangerous" factor from (1), the non-diagonal ion pressure due to reflected and gyrating ions, is expected to be small. Based upon a sampling of bow shock observations made by ISEE-1 and -2, we conclude that this technique is a satisfactory alternative when two-spacecraft comparisons are not feasible. For slightly-supercritical shocks (where a small foot structure is observed), estimates of scale size based on the non-collinear component/ramp-width relationship and based on the specular-ion-reflection/foot-length relationship agree. However, the technique outlined here is most useful for laminar shocks, a variety of shocks for which no independent technique for determining scale size was previously available.

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