Scaling law test and two predictions of planetary magnetic moments

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The scaling law for the strength of planetary magnetic dynamos, originally derived from a magnetohydrodynamic (MHD) theory\(^1\), has recently been independently re-derived using simple dimensional analysis\(^2\). While dimensional analysis is certainly no replacement for a solution based on a physical mechanism, this analysis does lend more credence to the scaling law by the implicit suggestion that the form of the scaling law is not an artefact of the MHD solution. We show here that the scaling law holds quite well in ordering the three well determined planetary magnetic moments. The present meagre data suggest that the terrestrial magnetic moment is about twice the value expected when compared with the other planets. Assuming that Earth is anomalous, rather than Mercury and Jupiter, and that the scaling law is valid, we predict that Saturn will be found to have a moment of \(1.2 \times 10^{26} \) G cm\(^3\) and Io a moment of \(6.5 \times 10^{22} \) G cm\(^3\).

Expressed in terms of magnetic moments which are the observable parameters in planetary exploration, the scaling law becomes

\[
\frac{M_p}{M_E} = \left( \frac{\rho_p}{\rho_E} \right)^{1/2} \left( \frac{R_p^2}{R_E^2} \right) \left( \frac{\Omega_p}{\Omega_E} \right)
\]

where \(M\) is the magnetic moment, \(R\) is the radius of the core, \(\rho\) is the density of the core and \(\Omega\) is the spin rate. The subscript \(P\) refers to the planet, \(E\) to the Earth. For this scaling law to work, all planetary dynamos must have the same underlying physical mechanism. We note also that the strength of the energy source for the dynamo does not appear in this scaling law. In this model if the strength of the energy source is not sufficient, the dynamo does not regenerate and then stops. If it is regenerative, it assumes a particular strength given by the scaling law. This lack of consideration of the strength of the energy source is considered by many to be grounds for rejection of any scaling law.

In the scaling law \(\Omega\) is well known for all large bodies in the Solar System except for the outermost planets. However, there is much uncertainty in the core radius and density, especially for the inner planets whose chemical composition and thermal history are uncertain. Nevertheless, enough modelling has been done, and planetary measurements performed for us to be able to put this scaling law to a serious test.

The results of this test are shown in Fig. 1. For Mercury we have made the core density the same as that of the Earth and the limits on core radius to be 1,470–1,840 km (refs 3, 4). The range of the observed magnetic moment was taken to be from 2.4 \times 10^{22} \text{ G cm}^3 to 5 \times 10^{22} \text{ G cm}^3 (refs 5–7). For Venus we have taken a core density equal to that of the Earth and a fractional radius within 10% of that of the Earth. The magnetic moment has been taken to be less than 1 \times 10^{22} \text{ G cm}^3 (ref. 8). For the Earth we have taken a moment of 8 \times 10^{25} \text{ G cm}^3, a radius of 3,486 km, and a density of 11 g cm\(^{-3}\) (ref. 9). For Mars, we have taken a range of models with realistic core densities given by Okal and Anderson\(^10\) for the internal structure and the magnetic moment of Dolginov and coworkers\(^11\) as an upper limit\(^12\). For Jupiter we use the models of Stevenson and Salpeter and its stated uncertainties\(^13\) and the moment of Davis and Smith\(^14\).

The straight line through the Earth has a slope of unity. We would expect that those planets with operating dynamos would fall on this line. Clearly Jupiter and Mercury do not. However, they both fall on a slightly displaced line with unit slope. Thus relative to Jupiter and Mercury the Earth seems to be anomalously high. It might be said that since the three planets with clearly established dynamos do not fall on one line, then the scaling law has failed. However, for the sake of further discussion, we will assume that the scaling law works but that the Earth is anomalous.

Venus and Mars do not lie near either line which suggests that they do not have dynamos at present. If one were to detect unambiguously an intrinsic field at Venus or Mars due to an internal dynamo at or below these limits, this observation would necessitate reassessment of Busse’s dynamo theory.

For Saturn we have taken Slattery’s model\(^15\) and arbitrarily assumed ±6% for the uncertainty in core radius. The dot marks the moment inferred from radio observations\(^16\). This data point falls squarely between the jovian and terrestrial scaling lines. If we assume that the jovian scaling line is the proper scaling line with which to compare Saturn we would predict a core radius for Saturn of 33,000 km, much greater than the 28,000 km assumed by Slattery. If the modelling is correct, then the expected moment is \(1.2 \times 10^{26} \text{ G cm}^3\) corresponding to an equatorial surface field of 0.5 G. While it is difficult to judge which is the more accurate, the theoretical modelling of Saturn’s interior or the scaling of the radio emissions, we argue that as the source of the radio emissions is not understood for the Earth, Jupiter or Saturn, the modelling efforts are more accurate at present. Thus, our first prediction is that Pioneer 11 will measure a moment of about \(1.2 \times 10^{26} \text{ G cm}^3\). If the jovian modelling efforts can be of any guide we expect this estimate to be correct to about ±20%.

Finally let us examine Io. A prediction for Io has already been made by Neubauer\(^17\). He assumes an Fe–FeS eutectic core with a density of 5.3 g cm\(^{-3}\) and a core radius of 890 km corresponding to an average chondritic composition. Voyager pictures of Io show many active volcanoes suggesting that Io is molten internally. This in turn suggests Io has in fact differentiated and formed an iron core. Thus far the Voyager mission has not provided further constraints on the size of the core. However, it is clear that the satellite is losing sulphur. Thus we will also use a higher core density, 7 gm cm\(^{-3}\) (ref. 10) in our estimates of the expected moment. This expectation is indicated by the vertical line which crosses the Earth scaling law at a moment of \(1.3 \times 10^{23} \text{ G cm}^3\), very close to Neubauer’s prediction. The line crosses the Jupiter–Mercury line at 6.5 \times 10^{22} \text{ G cm}^3. This same moment has been inferred from observations of Io’s ionosphere.
and effects on particle motions by Kivelson and coworkers. Our second prediction, based on the adoption of the Jupiter-Mercury line as the best scaling line, is that Io has a moment of close to $6.5 \times 10^{22}$ G cm$^3$. The weak link in this prediction is the choice for the size of the core of Io.

If the above predictions are correct, we must seek an explanation for why the Earth’s field is anomalously high, as the scaling law does not involve the strength of the energy source. Perhaps the answer lies in the precessional torques applied to the Earth’s core by the Moon.

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*Note added in proof:* On 1 September 1979, Pioneer 11 flew by Saturn and detected a magnetic moment of $4.8 \times 10^{24}$ G cm$^3$ (E. J. Smith, personal communication). To be consistent with the Busse scaling law, the conducting core of Saturn would have to be 23,000 km in radius. Unless future analysis of the Pioneer 11 gravity data and a reassessment of the models of the interior structure are consistent with such a small core we must conclude that Busse’s scaling law is useful for predicting planetary moments only to within a factor of about 2.5.

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